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THE JOHNS HOPKINS UNIVERSITY
Department of Civil Engineering
BALTIMORE, MARYLAND
December, 1952

**THE STEADY MOTION OF A
SYMMETRICAL OBSTACLE
ALONG THE AXIS OF
A ROTATING FLUID**

By Robert R. Long

Technical Report No. 1



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THE OFFICE OF NAVAL RESEARCH

The research reported in this document was supported by the Office
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THE STEADY MOTION OF A SYMMETRICAL
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ABSTRACT

An investigation by G. I. Taylor of the steady motion of an obstacle along the axis of a rotating fluid, is extended in the following paper. It is shown that Taylor's particular solution is just one of an infinity of functions comprising the general solution.

The theory is applied to motions in a rotating cylinder of fluid. A critical Rossby number is derived, below which the flow around the obstacle is wave-like. When the Rossby number is greater than the critical value, the flow consists only of a local perturbation that dies out rapidly on both sides of the obstacle. Various other critical numbers exist, below which additional modes of oscillation become dynamically possible.

An experiment was designed to test the theoretical results of this paper. An obstacle was moved along the axis of a long cylinder of rotating water. The resulting flow patterns were observed visually and photographically. The three-dimensional wave motions which occurred in the experiment were unquestionably the same as those in the theoretical solution.

THE STEADY MOTION OF A SYMMETRICAL OBSTACLE ALONG
THE AXIS OF A ROTATING FLUID

1. Introduction

In a series of papers written a number of years ago, Taylor (1920, 1922, 1923), Proudman (1916), and Grace (1922, 1923, 1924, 1926) investigated the problem of the motion of a perfect, rotating fluid around obstacles (especially spheres). This work has acquired renewed interest in the recent work of Morgan (1951) and Stewartson (1952).

With the exception of Taylor, the problem is of academic interest alone to the above authors. Indeed, it represents a fascinating challenge for the extension of the investigations of classical hydrodynamics to an area that is almost unexplored. If the problem had no other importance, the present paper should, no doubt, have been submitted to the Proceedings of the Royal Society of London in which almost all of the above researches have appeared. It is almost obvious that this is not, or should not be, the case. Certainly all meteorologists are agreed that the moderate and large-scale motions of the atmosphere are profoundly influenced by the vorticity of the earth's rotation. It would seem to be evident, then, that efforts to understand these phenomena should begin with studies of flow of a rotating fluid around elementary bodies. The motion of an irrotational liquid around a sphere is discussed in an early chapter of Lamb's Hydrodynamics, and Stokes' solution for viscous flow around a sphere was one of the first investigations based on the Navier-Stokes equations of motion.

It is unfortunate that basic investigations of a rotating fluid are infrequent in meteorological literature. The resulting lack of knowledge of the most fundamental effects of rotation on fluid motions has been a distinct handicap in current efforts to produce laboratory models of atmospheric phenomena. These difficulties are discussed in a series of papers (Long, 1951, Long, 1952a, Fultz and Long, 1951).

One of the most interesting of these basic problems to a meteorologist is the motion of an obstacle in a rotating fluid in a direction perpendicular to the axis of rotation. This motion would have analogies to the effect of mountain barriers on the ever-present zonal currents in the earth's atmosphere.

Efforts in this direction have not proved to be particularly successful, however, since the analysis is very complicated. The steady motion of a sphere along the axis of a rotating fluid, as analysed by Taylor (1922), is simpler and his results were the only ones that did not require the assumption of infinitesimally small motions. As shown below, Taylor's solution is only one of an infinite sum of functions which comprise the general solution, and he was unable, therefore, to obtain a physically realizable flow pattern. In the present paper, section 2 is devoted to a further theoretical discussion of this problem. An experiment to test the theory is described in section 3.

2. General solution

Figure 1 is a sketch of the coordinate system used in the theoretical development of the flow around a symmetrical obstacle moving steadily along the axis of a rotating fluid. The x-axis is along the axis of rotation, and velocities in this direction are denoted by u . The component of the velocity outward is v , and w is the component of the absolute velocity tangent to circles with centers on the x-axis. The center of the coordinate system is assumed to move with the obstacle at a speed u_0 , so that the flow with respect to this frame is steady. Obviously, this requires the additional assumption that the fluid extends to infinity in both directions along the axis of rotation.

The velocity components are

$$u = \frac{dx}{dt}, \quad v = \frac{dr}{dt}, \quad w = r \frac{d\phi}{dt}, \quad (1)$$

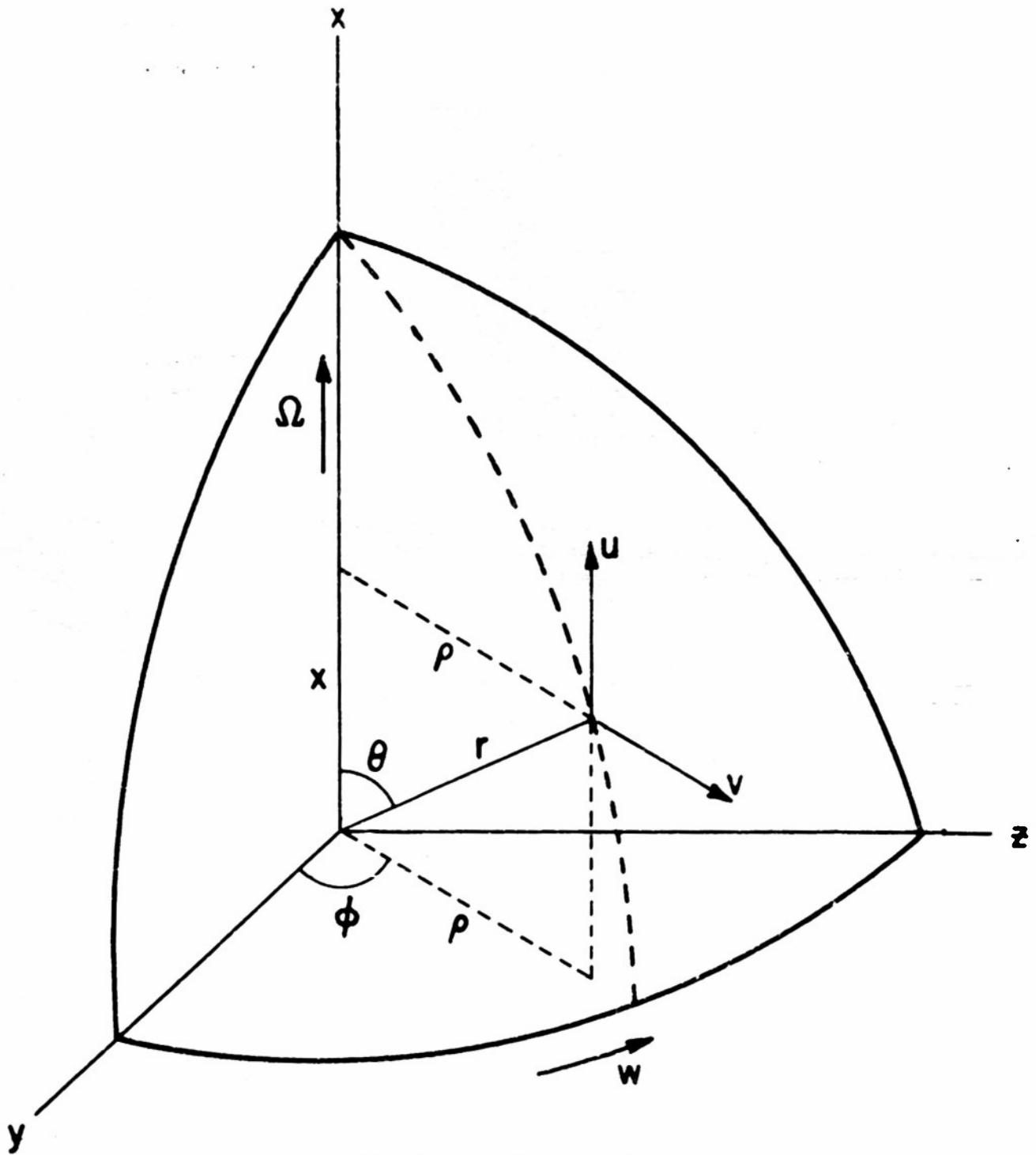


Fig. 1—Coordinate system.

and the individual time derivative is

$$\frac{d}{dt} = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial \rho} \quad (2)$$

In (2) the partial derivative with respect to time is missing because of the assumed steady state. The derivative with respect to ϕ is also zero since the motion will be symmetrical about the x-axis¹. The equations of motion and continuity for an incompressible, homogeneous fluid are

$$\frac{dv}{dt} - \frac{w^2}{\rho} = -\frac{\partial}{\partial \rho} \left(\frac{\rho}{q} + \chi \right), \quad (3)$$

$$\frac{du}{dt} = -\frac{\partial}{\partial x} \left(\frac{\rho}{q} + \chi \right), \quad (4)$$

$$\frac{dw}{dt} + \frac{wv}{\rho} = 0, \quad (5)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial \rho} + \frac{v}{\rho} = 0, \quad (6)$$

where q is the (constant) density, and χ is the potential of the external forces (gravity). The equation of continuity, (6), reveals that we may express the velocity components, u , v , by a stream function, $\psi(x, \rho)$, as follows:

$$u = -\frac{1}{\rho} \frac{\partial \psi}{\partial \rho}, \quad v = \frac{1}{\rho} \frac{\partial \psi}{\partial x} \quad (7)$$

Equations (2) and (7) show that, if a quantity is individually conserved during the motion, it is then a function only of the stream function, ψ . Combining terms

¹ This may be inferred from the fact that the equations of motion and continuity, and the kinematic boundary conditions are invariant for transformations of ϕ to $\phi' + \text{const}$. This application of group theory (Birkhoff, 1950) is almost trivial in this case.

in (5) we find that

$$w\rho = g(\psi) \quad (8)$$

We determine the unknown function, $g(\psi)$, by assuming that at a sufficiently great distance ahead of the obstacle the perturbed motion vanishes. This requires that $g(\psi) = \Omega \rho_o^2$, where ρ_o is the undisturbed distance of the stream line from the axis of rotation. Similarly, from (7), $\psi = -u_o \rho_o^2/2$ in a Lagrangian sense. Combining these results,

$$w_i = -\frac{\sigma\psi}{\rho}, \quad \sigma = \frac{2\Omega}{u_o} \quad (9)$$

If we define the following quantity:

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial \rho}, \quad (10)$$

and eliminate the right hand sides of (3) and (4) by cross differentiation, we obtain

$$\frac{d}{dt} \left(\frac{\xi}{\rho} + \frac{\sigma^2 \psi}{\rho^2} \right) = 0, \quad (11)$$

or

$$\frac{\xi}{\rho} + \frac{\sigma^2 \psi}{\rho^2} = H(\psi). \quad (12)$$

Evaluating $H(\psi)$ in a manner similar to the determination of $g(\psi)$ in (8),

$$\frac{\xi}{\rho} + \sigma^2 \left(\frac{\psi}{\rho^2} + \frac{u_o}{2} \right) = 0 \quad (13)$$

The vorticity component, ξ , may be expressed in terms of ψ by using (7) and (10).

The following two equations then replace the original set (3)-(6):

$$\rho w = -\sigma \psi \quad (14)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial \rho^2} - \frac{1}{\rho} \frac{\partial \psi}{\partial \rho} + \sigma^2 \psi = -\frac{\sigma^2 u_o \rho^2}{2} \quad (15)$$

The general solution is

$$-\rho \frac{w}{\sigma} = \psi = -\frac{u_0 \rho^2}{2} + \rho \sum_k (A_k \cos kx + B_k \sin kx) J_1[(\sigma^2 - k^2)^{\frac{1}{2}} \rho] \quad (16)$$

In this equation J_1 is a Bessel function of the first kind. The Neumann function becomes infinite on the axis of rotation ($\rho = 0$) and hence is omitted.

Equation (15) may also be expressed in spherical coordinates by the relations (see fig 1),

$$x = r \cos \theta, \quad \rho = r \sin \theta. \quad (17)$$

The corresponding form of the solution is

$$\psi = -\frac{u_0 r^2 \sin^2 \theta}{2} + (or)^{\frac{1}{2}} \sum_{n=1}^{\infty} \sin \theta P_n'(\cos \theta) [A_n J_{n+\frac{1}{2}}(or) + B_n J_{n-\frac{1}{2}}(or)] \quad (18)$$

The Bessel functions are those of the half-odd order and, as is well known, are expressible in terms of elementary functions (Margenau and Murphy, 1943).

$P_n^1(\cos \theta)$ is an associated Legendre polynomial. The particular solution for $n = 1$ is the solution obtained by Taylor.

Taylor experienced a difficulty in applying his one solution to the case of the flow around a sphere in a rotating fluid extending to infinity in all directions. The kinematic condition at the obstacle and the condition that the perturbation vanish at infinity determined only one of the two constants at his disposal, and he was left with an indeterminate problem. The fact that the present paper adds

a new infinity of solutions would seem to be a further complication. Experience with other problems of a similar nature indicates that this is not the case (Rayleigh 1883, Lyra 1943, Long 1952b). In these references fluid systems are considered in which obstacles set up waves in a current passing over or around them. In all cases the same indeterminacy is found. The first investigation of this phenomenon was made by Rayleigh, who discovered that the introduction of viscosity in the equations of motion removed the indeterminacy. Then, letting the frictional coefficient tend to zero, the solution is unique and involves a complete damping of the upstream waves. The resulting asymptotic solution, called the "practical solution" by Lamb (1932), is the one that is realized in actual experiments (Bakhmeteff, 1932). Applying this to (18), all the remaining constants would be required to annul the "upstream" oscillations. The experimental work described in this paper verifies that no waves occur in advance of the obstacle (see fig 9).

Although (16) or (18) represents the general solution for the motion considered in this paper, there are great difficulties in finding a solution that satisfies all the conditions in a given special case. Even for a sphere moving in an unlimited fluid the problem of choosing the coefficients of the Bessel functions in (18) so as to annul the "upstream" waves, seems prohibitively complicated. If the cylinder has a finite radius, the difficulties are even greater. Yet the complete solution for a simple type of obstacle is greatly to be desired in order to ascertain its behavior as the parameter σ tends to infinity (i.e. u_0 tends to zero). Taylor's experiments and those reported in this paper show that an obstacle, moving very slowly along the axis, pushes a column of fluid ahead of it with little disturbance in the fluid outside. This is one of the peculiarities in the behavior of a rotating liquid and should be carefully studied.

A fairly good idea of the motion in a cylinder of radius b may be inferred from (16). The kinematic condition at the outer wall requires that the Bessel function

have a zero there, so that

$$\psi = -\frac{u_0 \rho^2}{2} + \rho \sum_{n=1}^{\infty} (A_n \cosh k_n x + B_n \sinh k_n x) J_1(z_n \frac{\rho}{b}) \quad (19)$$

$$+ \rho \sum_{n=n_1+1}^{\infty} (A_n \cosh k_n x + B_n \sinh k_n x) J_1(z_n \frac{\rho}{b}),$$

where z_n are the values of the arguments of $J_1(z)$ when this function is zero. z_1 is 3.8317, for example, and the others are given in Jahneke and Emde (1945). The quantity k_n is

$$k_n = \left| \left(\sigma^2 - \frac{z_n^2}{b^2} \right)^{\frac{1}{2}} \right|, \quad (20)$$

and n_1 is the last integer for which $\sigma > z_n/b$. Since each of the functions of x in the second summation of (19) are non-oscillatory the sum must decrease steadily to zero in both directions. This part of the solution, therefore, represents a local perturbation near the obstacle that presumably dies out rapidly "upstream" and "downstream." Furthermore, if $z_1 > \sigma b$, the first summation in (19) is absent and the obstacle produces only a local disturbance. If $z_1 < \sigma b$, waves will be produced of length,

$$\lambda_n = \frac{2\pi b}{(\sigma^2 b^2 - z_n^2)^{\frac{1}{2}}} \quad (21)$$

*It is of interest to express the critical condition, dividing the oscillatory motion from the non-oscillatory type of flow, in terms of the Rossby number, $R_0 = u_0/2 \Omega b$. It is

$$R_0 = \frac{1}{3.8317} = .261. \quad (22)$$

In terms of this parameter the first mode of oscillation has a length

$$\lambda = \frac{2\pi b}{(R_0^2 - 14.68)^{1/2}} \quad (23)$$

It is not surprising that for large Rossby numbers the obstacle should produce only a local perturbation. An infinitely large R_0 corresponds to a vanishing, basic angular velocity. The motion, therefore, should approach potential flow as $R_0 \rightarrow \infty$. It is obvious from the differential equation (15) that it does so.

The waves that occur when $R_0 < .261$ are the subject of the experimental study described in the next section. Without solving for the motion around a given obstacle we cannot tell how the waves combine to give the total wave pattern. If $n_1 = 1$, however, only one mode can exist, and, sufficiently far behind the obstacle, this wave should represent the only sensible disturbed motion. Using the second zero of the function J_1 , the one-wave system should occur in the range,

$$.142 < R_0 < .261. \quad (24)$$

A plot of the wave length vs. R_0 is shown in fig 2.

The above discussion has referred only to the component of the velocity in planes through the axis of the cylinder. The component, w , normal to these planes is given analytically in (14) or (16). Physically the meaning of (14) is that each fluid ring (centered on the x-axis) conserves its angular momentum during the motion. Therefore, the ring slows down when expanding, and speeds up when contracting, i.e. when v is positive or negative respectively. The result is a three-dimensional oscillation which has a phase difference of one-quarter wave length when projected on planes at right angles.

3. Experiments

Taylor's investigation of the problem of this paper included an experiment with a light sphere pulled along the axis of a rotating cylinder of water. His results were very meager, however. No definite wave motion was seen, probably

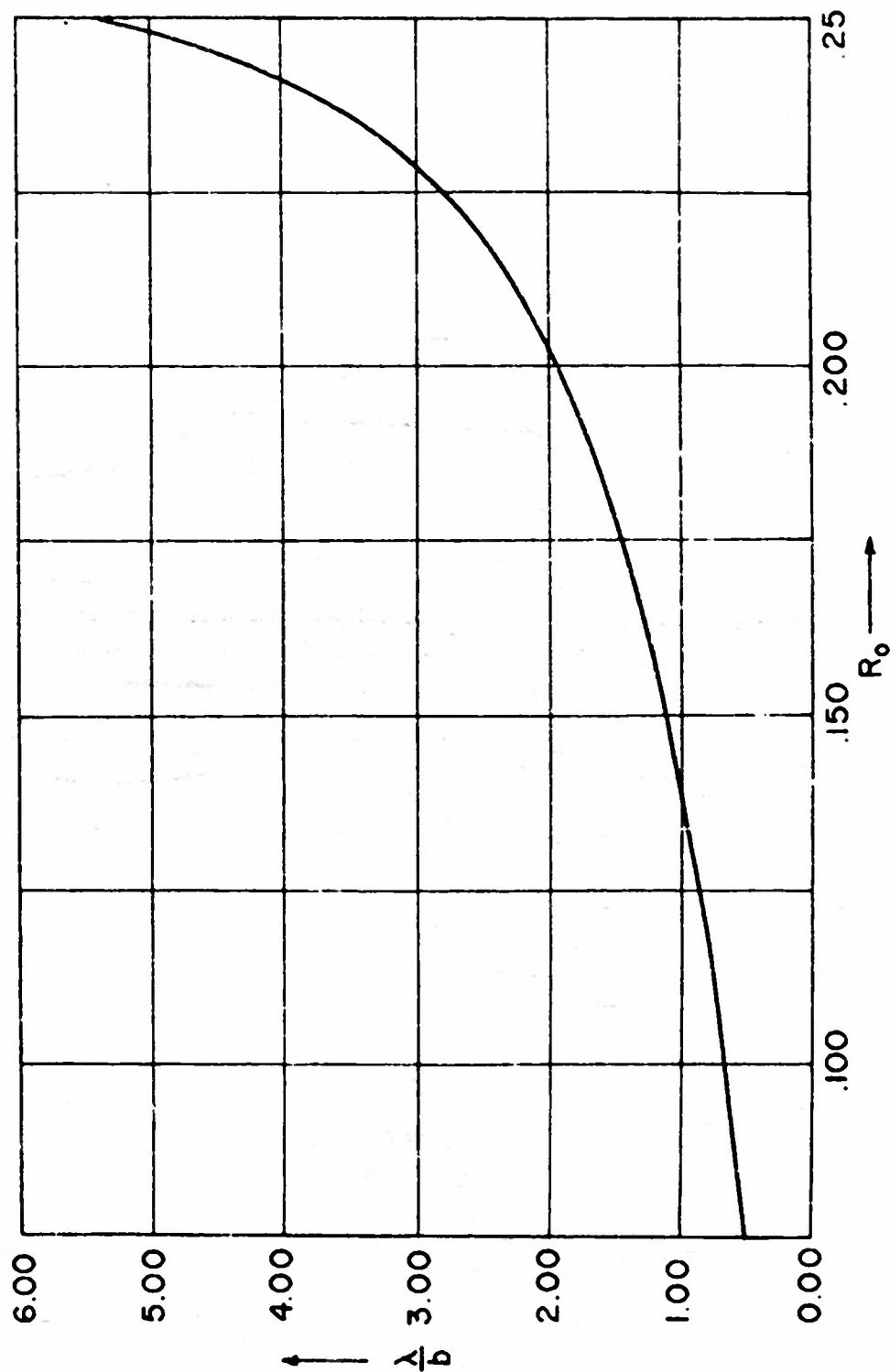


Fig. 2--Plot of equation (23).

because he used such a small obstacle that the waves were not excited by the forced perturbation. In view of this deficiency, and of the new theoretical findings in section 2, it seemed advisable to make a further experimental study.

The equipment constructed for this work is shown in fig 3. The long cylinder is 90 cm in length and has an inner diameter of 29 cm. The obstacle, shown in position at the axis, is a sphere with an attached cone to form a tapered trailing edge. The purpose of this is to minimize the effect of boundary layer separation. The spherical head has a radius of 3.8 cm and the entire obstacle has an overall length of 11.2 cm. The cylinder is mounted on a turntable and rotated at a uniform speed (usually 10 rpm). The water assumes the angular velocity of the vessel within a period of 15-20 minutes. The obstacle is lowered by a motor drive into the water and along the axis of the cylinder. A scale, parallel to this axis, is placed just outside of the cylinder to facilitate the measurement of wave lengths. In view of the dimensions used in fig 2, this length scale is in units of b , the inside radius of the vessel (14.5 cm).

The problem of observing the flow in this experiment is complicated by the basic rotation and the fact that the motions are three-dimensional. The most satisfactory tracer was a purple dye trail left by a dissolving pellet of potassium permanganate dropped into the water just before the descent of the obstacle along the axis. These pellets, when dropped in a cluster, leave a group of dye filaments which remain parallel to the axis of rotation until deformed by the passage of the obstacle. Since the filaments are material lines they are the stream lines of the motion relative to a coordinate system moving with the obstacle and rotating with the cylinder. Photographing these lines necessarily projects the pattern on the plane of the camera but the wave length is preserved by this projection. The three-dimensionality of the motion is recorded by taking simultaneous pictures with two cameras at 90 degrees. It did not seem to be necessary, however, to include both photographs in the figures of this paper.

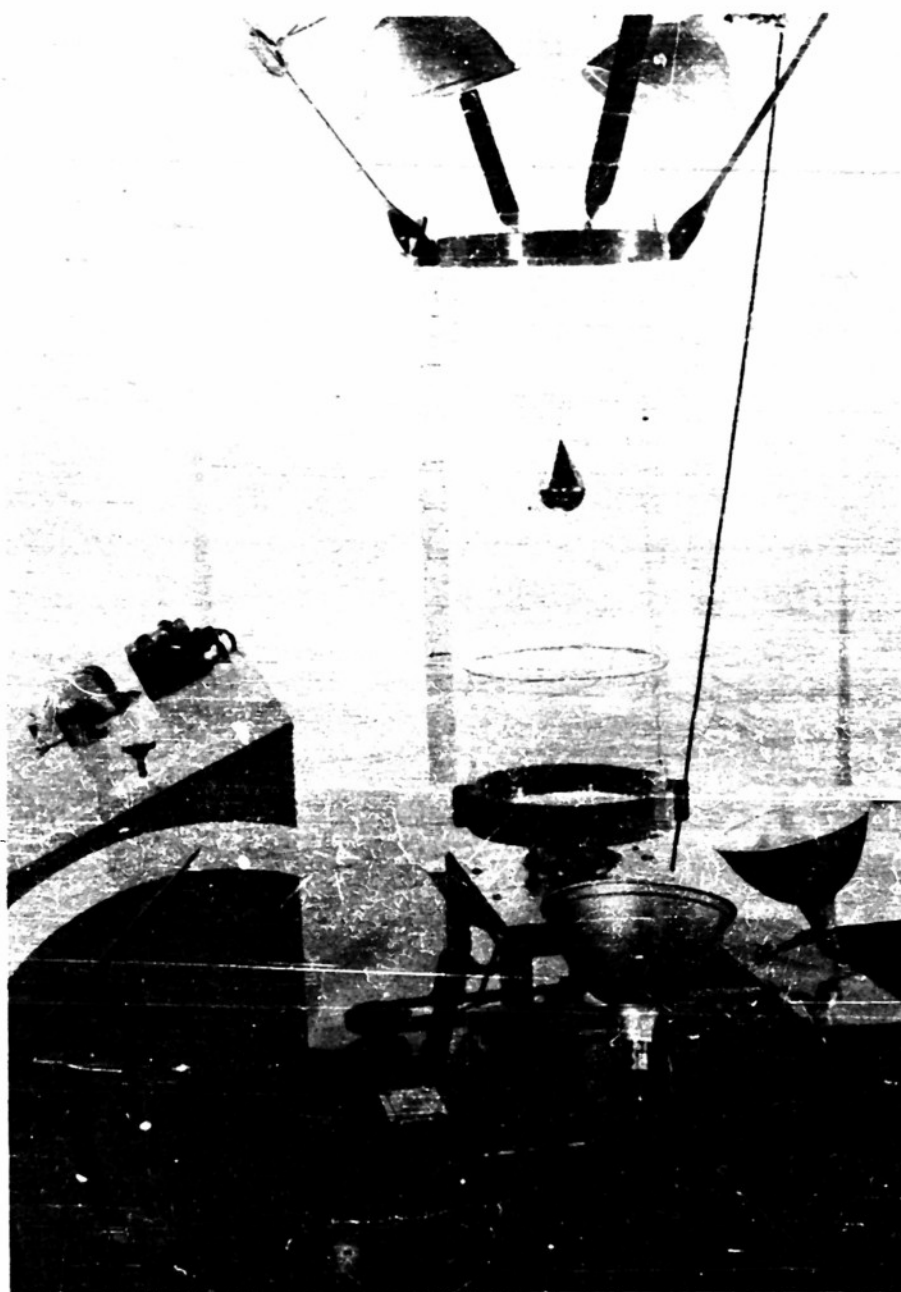


Fig. 3—Experimental equipment. The long, glass cylinder is filled with water and rotated at a constant angular velocity. The obstacle is lowered at a uniform speed along the axis of rotation.

The behavior of the dye pellets, when dropping through the water, is somewhat erratic and there is considerable difficulty in following a given filament in a photograph over an entire wave length. This lends some uncertainty to the determination of the wave length in most of the photographs.

Visual observations of the motion around the obstacle confirmed immediately the preliminary theoretical deductions in section 2 of this paper. As the obstacle approached a fluid ring, centered on the axis of rotation, it expanded to permit its passage around the barrier. The requirement for conservation of angular momentum caused it to lose some of its basic angular velocity as shown by the twisting of the dye filaments in a direction opposite to the basic rotation. In the lee of the obstacle the fluid ring performed a series of oscillations, expanding and contracting and losing and gaining some of its spin around its axis.

Two complications have prevented a detailed comparison of observed and theoretical wave lengths. In the first place, the theoretical findings were based on the assumption of an infinitely long cylinder. In the experiment the wave length is an appreciable fraction of the cylinder length. It appears from the observations that this causes a marked shortening of the wave nearest the free surface. Secondly, in the vicinity of the obstacle a local perturbation is added to the free wave motion. This causes the first measured wave length in the lee of the barrier to be somewhat longer than the theoretical length. These two factors make the measurement of the free wave length a subjective matter, especially for the longer waves. The experiments leave little doubt, however, that the observed waves are the same as the theoretical oscillations. This is shown rather convincingly in the photographs.

Figure 4 is a photograph of the wave motion at a low Rossby number (0.083). In this experiment, as well as in figs 5 and 6, R_0 is in a range where more than one mode of oscillation is possible. Nevertheless, the wave lengths approach those given by equation (23). If the other modes exist they must have very small

amplitudes. Figures 7 and 8 have higher Rossby numbers and the waves are longer. Wave motion is still evident at Rossby numbers above 0.171 but they are so long that it is impossible to find one that is not affected by the local perturbation at the obstacle or by the presence of the free surface.

Figure 9 is a photograph showing the motion well ahead of the obstacle. As in all experiments, there is no wave motion "upstream."

Finally brief mention may be made of the flow at very low Rossby numbers. If R_0 is less than about 0.03 the experiments indicate that an obstacle pushes a column of fluid ahead of it as it moves along the axis. The radius of this column is indefinite but is somewhat smaller than the radius of the obstacle so that some fluid still flows around the barrier. When the fluid rings converge on the lee side a rather strong cyclonic vortex is formed there. Waves of small amplitude and very short wave length can still be seen on the "downstream" side of the obstacle.



Fig. 4—Motion around an obstacle moving along the axis of rotation of a long, rotating cylinder of water. The Rossby number of this experiment is 0.683. If the first mode of oscillation is the only one present, the theoretical length of the waves is $0.55b$. The waves between the arrows have successive lengths of, approximately, $0.69b$, $0.63b$, $0.61b$, $0.57b$, $0.53b$.



Fig. 5—Wave motion around an obstacle in a long, rotating cylinder. The Rossby number in this experiment is 0.101. The theoretical wave length of the first mode of oscillation is $0.69b$. The waves between the arrows have successive lengths of $0.70b$, $0.69b$, $0.69b$.



Fig. 6—Wave motion around an obstacle in a long, rotating cylinder. The Rossby number in this experiment is 0.138. The theoretical wave length of the first mode of oscillation is $1.02b$. The waves between the arrows have successive lengths of $1.25b$ and $1.02b$.



Fig. 7—Wave motion around an obstacle in a long, rotating cylinder. The Rossby number in this experiment is 0.153. The theoretical wave length is $1.19b$. The waves between the arrows have successive lengths of $1.34b$ and $1.17b$.

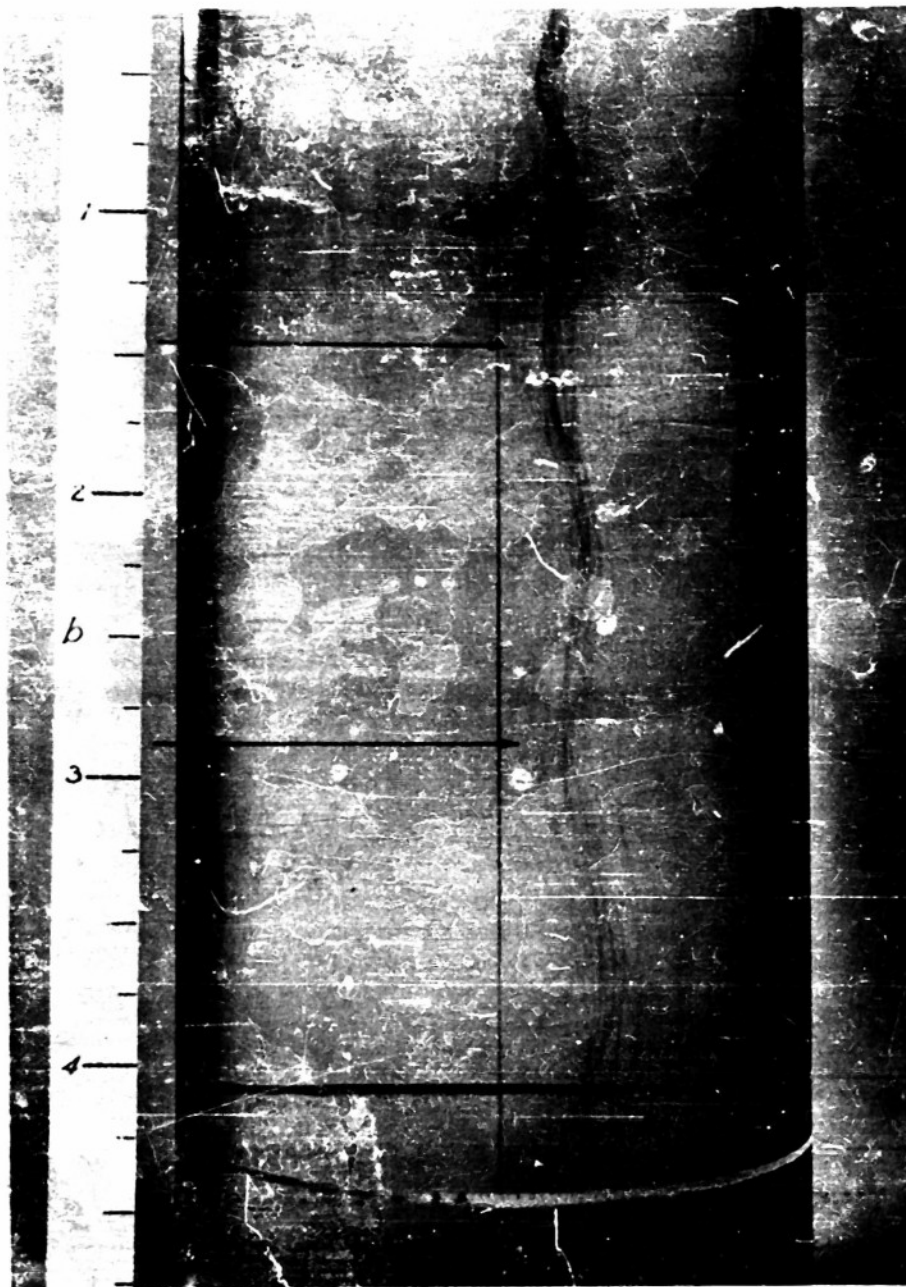


Fig. 8--Wave motion in a long, rotating cylinder of water. The Rossby number in this experiment is 0.171. The theoretical wave length is $1.42b$. The length of the wave between the arrows is close to this value. The shortening of the waves near the free surface is quite evident.



Fig. 9—Motion around an obstacle in a long, rotating cylinder of water. This photograph shows that no wave motion occurs ahead of the obstacle.

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